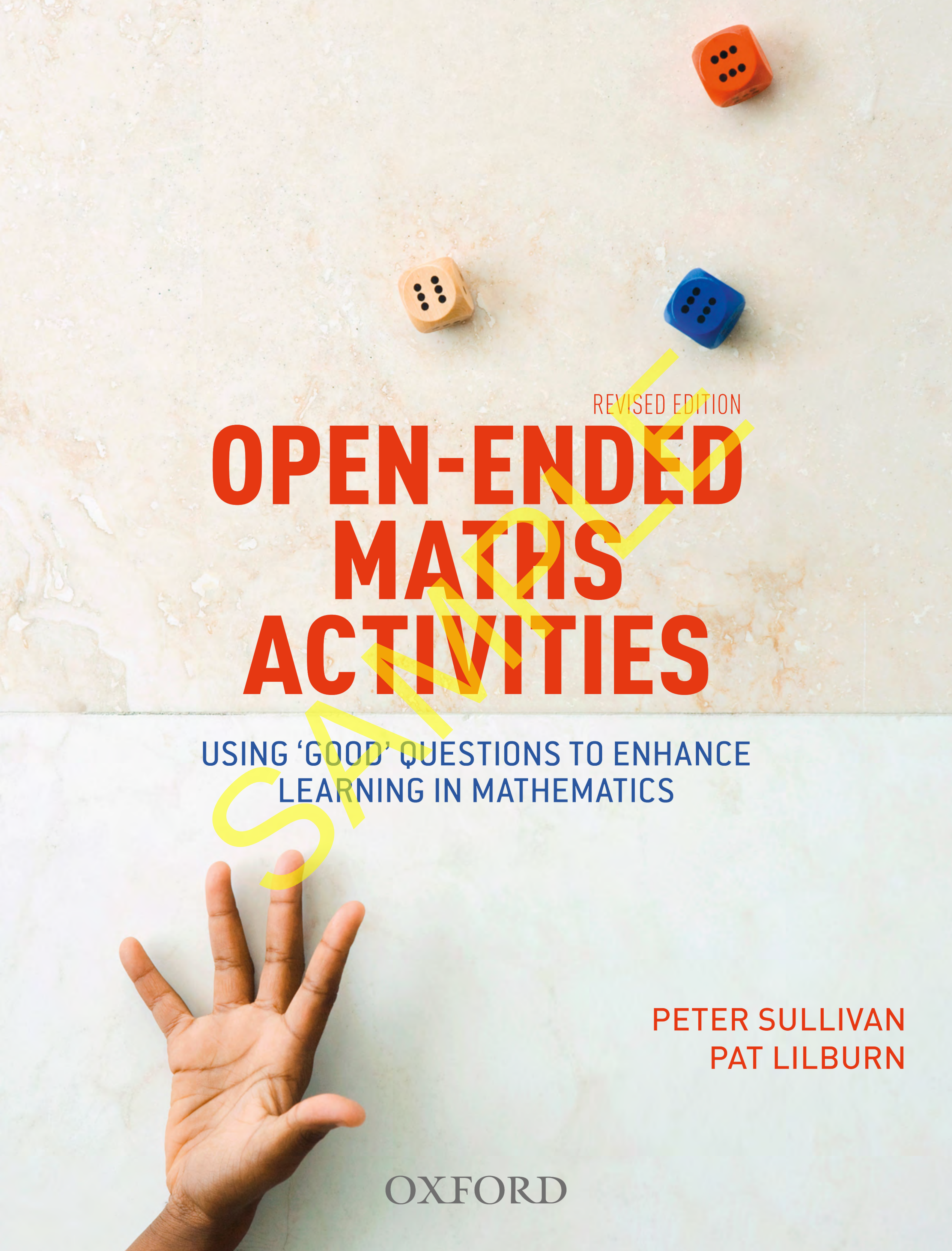

REVISED EDITION

OPEN-ENDED MATHS ACTIVITIES

USING 'GOOD' QUESTIONS TO ENHANCE LEARNING IN MATHEMATICS

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CONTENTS

PART

ONE

THE IMPORTANCE OF QUESTIONING

During the course of a normal school day, teachers ask many questions. In fact, something like 60% of the things said by teachers are questions and most of these are not planned.

One way of categorising questions is to describe them as either 'open' or 'closed'. Closed questions are those that simply require an answer or a response to be given from memory, such as a description of a situation or object or the reproduction of a skill. Open questions are those that require a student to think more deeply and to give a response which involves more than recalling a fact or reproducing a skill.

Teachers are usually skilled at asking open questions in learning areas such as language or social studies. For example, teachers often ask students to interpret situations or justify opinions. However, in mathematics lessons closed questions are much more common.

Questions which encourage students to do more than recall known facts have the potential to stimulate higher levels of thinking. To emphasise problem solving, application, and the development of a variety of thinking skills it is vital that we pay more attention to improving our questioning in mathematics lessons. Teachers should use questions which develop their students' higher levels of thinking.

Open-Ended Maths Activities looks in more detail at a particular type of open question which we call a 'good' question. If, as Sullivan and Clarke[1] say, 'our goals of education are for our students to think, to learn, to analyse, to criticise and to be able to solve unfamiliar problems', then it follows that 'good' questions should be part of the instructional repertoire of all teachers of mathematics.

In this book we describe the features of 'good' questions, show how to create 'good' questions, give some practical ideas for using them in your classroom, discuss how to assess 'good' questions and provide many 'good' questions that you can use in your mathematics program.

WHAT ARE 'GOOD' QUESTIONS?

Let us have a closer look at what makes a 'good' question.

There are three main features of 'good' questions.

1. They require more than remembering a fact or reproducing a skill.
2. Students can learn by answering the questions, and the teacher learns about each student from the attempt.
3. There may be several acceptable answers.

This section explains these features in more detail.

1 MORE THAN REMEMBERING

Bloom's[2] well-known taxonomy described six levels of thinking: knowledge, comprehension, application, analysis, synthesis and evaluation. Much mathematics teaching simply requires students to recall some knowledge or reproduce a skill — activities that fall under 'knowledge', the lowest level according to Bloom.

Few of the activities which appear in major mathematics texts require much analysis, synthesis or evaluation. That is, students are not required to make conjectures, follow arguments or to comment on results.

Consider the following example. A particular year 6 student, Jane, had just finished a unit on measurement where she had been asked to calculate area and perimeter from diagrams of

1 Sullivan, P. and Clarke, D. *Communication in the Classroom*, Geelong, Deakin University Press, 1990, p. 13

2 Bloom, B. S. *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook I: Cognitive Domain*, New York, David McKay, 1956.

rectangles with the dimensions marked. She was able to complete these correctly, and the teacher assumed from this that Jane understood the concepts of area and perimeter. However, when she was asked the following 'good' question she claimed that she could not do it because there was not enough information given. *I want to make a garden in the shape of a rectangle. I have 30 metres of fence for my garden. What might be the area of the garden?*

To find an answer to this Jane needed to think about the constraints that a perimeter of 30 metres places on the lengths of the sides of the rectangle, as well as thinking about the area. She needed to use higher order reasoning skills since she had to consider the relationship of area and perimeter to find possible whole number answers that could range from 14×1 ($14\ m^2$) to 7×8 ($56\ m^2$). This certainly required her to do more than remember a fact or reproduce a skill. It required comprehension of the task, application of the concepts and appropriate skills, and analysis and some synthesis of the two major concepts involved.

Through further probing, this question allowed the teacher to see that Jane had little appreciation of perimeter as the distance around a region, and no concept of area as covering. She had learned to answer routine exercises without fully understanding the concepts.

Another example of closed questions commonly found in textbooks is from the topic of averages. A typical question looks something like *What is the average of 6, 7, 5, 8 and 4?* This mainly requires students to recall a technique. That is, add the numbers and divide by how many there are — in this case five. However, if this question was rephrased in the form of a 'good' question it would look something like *The average of five numbers is 6. What might the numbers be?* or *After five games, a bowler has averaged six wickets per game. What might be the number of wickets he took in each game?*

These questions require a different level of thinking and a different type of understanding of the topic of averages to be able to give an answer. Students need to comprehend and analyse the task. They must have a clear idea of the concept of average and either use the principle that the scores are evenly placed about the average or that the total of the scores is 30 (i.e. 5×6) as the basis of their response. It most definitely requires more than remembering.

2 STUDENTS LEARN BY DOING THE TASK AND TEACHERS LEARN FROM THE STUDENTS' ATTEMPTS

'Good' questions are particularly suitable for this because they have the potential to make students more aware of what they do know and what they do not know. That is, students can become aware of where their understanding is incomplete. The earlier question about area and perimeter showed that by thinking about area and perimeter together the student is made aware of the fact that the area can change even though the perimeter is fixed. The very act of trying to complete the question can help students gain a better understanding of the concepts involved. The manner in which some students went about answering the following question illustrates this point.

John and Maria each measured the length of the basketball court. John said that it was 20 rulers long, and Maria said that it was $19\frac{1}{2}$ rulers long. How could this happen?

Some upper primary students were asked to discuss this question in groups. They suggested a variety of plausible explanations and were then asked to suggest what they need to think about when measuring length. Their list included the need to:

- agree on levels of accuracy
- agree on where to start and finish, and the importance of starting at the zero on the ruler
- avoid overlap at the ends of the rulers
- avoid spaces between the rulers
- measure the shortest distance in a straight line.

By answering the question, the students established for themselves these essential aspects of measurement, and thus learnt by doing the task.

As we have discussed, the way students respond to 'good' questions can also show the teacher if they understand the concept and can give a clear indication of where further work is needed. If Jane's teacher had not presented her with the 'good' question she would have thought Jane totally understood the concepts of area and perimeter. In the above example, the teacher could see that the students did understand how to use an instrument to measure accurately. Thus we can see that 'good' questions are useful as assessment tools too.

3 SEVERAL ACCEPTABLE ANSWERS

Many of the questions teachers ask, especially during mathematics lessons, have only one correct answer. Such questions are perfectly acceptable, but there are many other questions that have more than one possible answer and teachers should make a point of asking these too. Each of the 'good' questions that we have already looked at has several possible answers. Because of this, these questions foster higher level thinking because they encourage students to develop their problem-solving expertise at the same time as they are acquiring mathematical skills.

There are different levels of sophistication at which individual students might respond. It is characteristic of such 'good' questions that each student can make a valid response that reflects the extent of their understanding. Since correct answers can be given at a number of levels, such tasks are particularly appropriate for mixed ability classes. Students who respond quickly at a superficial level can be asked to look for alternative or more general solutions. Other students will recognise these alternatives and search for a general solution.

If we think back to the earlier question on the area of the garden, there is a range of acceptable whole number answers ($14 \times 1, 13 \times 2, 12 \times 3 \ldots 8 \times 7$). Students could be asked to find the largest or smallest garden possible. They could be asked to describe all possible rectangles. Other students will be interested in exploring answers other than those that involve only whole numbers, for example, 12.5 m × 2.5 m. It is the openness of the task that provides this richness. The existence of several acceptable answers stimulates the higher level thinking and the problem solving.

In this section we have looked more closely at the three features that categorise 'good' questions. We have seen that the quality of learning is related both to the tasks given to students and to the quality of questions the teacher asks. Students can learn mathematics better if they work on questions or tasks that require more than recall of information, and from which they can learn by the act of answering the question, and that allow for a range of possible answers.

'Good' questions possess these features and therefore should be regarded as an important teaching tool for teachers to develop. The next section shows two ways to construct your own 'good' questions.

HOW TO CREATE 'GOOD' QUESTIONS

'Good' questions can be used as the basis for an entire lesson either as a lesson that stands alone or as part of a unit of work. It is possible to make up your own 'good' questions for any topic and any year level. The important thing is to plan the questions in advance, as creating them is not something that can be done 'on your feet'.

When you first start using 'good' questions you might find helpful the collection of questions in Part Three, 'Good' questions to use in maths lessons. After a while you will want to create 'good' questions for yourself. Detailed here on pages 5 and 6 are two methods that can be used to construct 'good' questions. The one you use is a matter of personal preference.

METHOD 1: WORKING BACKWARDS

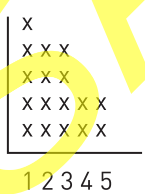
This is a three-step process.

Step 1 Identify a topic.
Step 2 Think of a closed question and write down the answer.
Step 3 Make up a question that includes (or addresses) the answer.

For example:

Step 1 The topic for tomorrow is averages.
Step 2 The closed question might be *The children in the Smith family are aged 3, 8, 9, 10 and 15. What is their average age? The answer is 9.*
Step 3 The 'good' question could be *There are five children in a family. Their average age is 9. How old might the children be?*

Some more examples of how this works are shown in the following table.

Step 1 Identify a topic.	Step 2 Think of an answer.	Step 3 Make up a question that includes the answer.
rounding	12 seconds	My coach said that I ran 100 m in about 12 seconds. What might the numbers on the stop-watch have been?
counting	4 chairs	I counted something in our room. There were exactly 4. What might I have counted?
area	6 cm^2	How many triangles can you draw each with an area of 6 cm^2?
fractions	$3\frac{1}{2}$	Two numbers are multiplied to give $3\frac{1}{2}$. What might the numbers be?
money	35 cents	I bought some things at a supermarket and got 35c change. What did I buy and how much did each item cost?
graphing		What could this be the graph of?

METHOD 2: ADAPTING A STANDARD QUESTION

This is also a three-step process.

Step 1 Identify a topic.
Step 2 Think of a standard question.
Step 3 Adapt it to make a 'good' question.

For example:

Step 1 The topic for tomorrow is measuring length using informal units.
Step 2 A standard exercise might be *What is the length of your table measured in handspans?*
Step 3 The 'good' question could be *Can you find an object that is three handspans long?*

Some more examples of how this works are shown in the following table.

Step 1 Identify a topic.	Step 2 Think of a standard question.	Step 3 Adapt it to make a 'good' question.
shape	What is a square?	How many things can you write about this square?
addition	337 + 456 =	On a train trip I was working out some distances. I spilt some soft drink on my paper and some numbers disappeared. My paper looked like 3 □ 7 + □ □ 6 7 9 □ What might the missing numbers be?
subtraction	731 − 256 =	Arrange the digits so that the difference is between 100 and 200.
time 	What is the time shown on this clock?	What is your favourite time of day? Show it on a clock.

The more experience you have with 'good' questions the more you will want to use them, and the easier it will become for you to make up your own. Refer to either or both of these methods until you feel confident.

USING 'GOOD' QUESTIONS IN YOUR CLASSROOM

Today's mathematics classrooms should be dynamic places where students are involved and engaged in their own learning. This can be achieved through activities that promote higher level thinking, co-operative effort, extension and communication.

We have seen that 'good' questions support these activities and are readily available for teachers to use. **The first part of this section describes generally how to use a 'good' question as the basis of a mathematics lesson. It sets out the important steps of the lesson, explains the roles of the teacher and students and advises how to overcome problems that could arise at each stage. The second part of this section takes you through each of the steps with a specific 'good' question.**

Before the start of a lesson it is necessary to choose or create a 'good' question. This should be aimed at the appropriate level for the students in your class. At first you might find the question you choose is too easy or too difficult, but keep practising because you will soon get the hang of it. Once you have chosen the question then the following steps should help you to use it with your class.

STEP 1: POSE THE 'GOOD' QUESTION

It is a good idea to have the question written on the board and as you ask the question refer to the words on the board. It is very important to make sure that all students know what the question is; do not assume they know it because it is on the board. You could even ask some students to repeat the question in their own words.

Allow some time for students to ask you about the meaning of the task. Explain the task to them if necessary but do not give any directions or suggestions on how to do it. This is for the students to work out for themselves.

STEP 2: STUDENTS WORK ON THE 'GOOD' QUESTION

When first using 'good' questions in your classroom it is better to let the students work in groups. This allows them to communicate their ideas to others in their group. This communication is an important part of learning. Working in groups can also assist those students who may have difficulty starting. If these students have to wait for the teacher, then organisational and attitudinal problems can arise.

If, once students start working, there are too many who cannot progress without teacher assistance then it might be necessary to stop them and have a whole class discussion to overcome the general concerns. If the concerns of each group, or individuals within each group, are all different then this is a sign that the question you have posed is too difficult for the class. If this happens, either make the question easier or suggest that the students represent the problem in some way, such as by using materials or drawing a diagram. A variety of concrete materials should always be available for students to select from. You could also decide to abandon the question altogether as unsuitable at this stage. If this happens do not worry, as it takes time and practice to choose appropriate 'good' questions. However, you will find 'good' questions to be worth the effort and perseverance. Ideally, you should plan in advance how to help students who may not be able to start on the question.

Once the groups are working, your task is to monitor their progress. If a group stops after giving one response, ask them to look for other possible answers. If they have found all possible answers ask them to describe all their answers. In this way they can experience the meaning of a general solution. You could also ask a related question to extend them. For example, a related question for the task *The stop-watch shows tenths of seconds. My coach said that I ran 100 m in about 12 seconds. What might the numbers on the stop-watch have been?* could be *What if the stop-watch showed hundredths?*

It is not vital that you wait until all groups have finished the task before initiating a discussion. They will all have answered the question to a degree. It is better to stop while students are still engaged with the question and interested in the task. This way they do not become distracted or need to be given additional work of a different type. You could give a five-minute warning before you stop groups so they have time to tie up the loose ends.

STEP 3: WHOLE CLASS REVIEW

This is an important phase. Ask the groups in turn to suggest responses and to explain their thinking. As each group does this write their responses on the board or, if this is not appropriate, display their model or diagram, making sure to give each group equal status. If a response is not suitable be supportive, but try to find out the cause of the error. As we saw earlier, 'good' questions can often make it easier for teachers to pinpoint exactly where their students are experiencing difficulty. Also, as students are explaining what they have done they often see the error for themselves anyway.

STEP 4: TEACHER SUMMARY

Usually, if the task is at an appropriate level, some of the students will make the main teaching point/s for you during the class review. Nevertheless, just because one or more students give a response does not mean that they all understand. Thus it is necessary to summarise the discussion for everyone, emphasising and explaining key points. Wherever possible do this using models and teaching aids. Because different people learn in different ways we need to use as wide a range of methods and materials as possible to model a situation. Also, make sure you relate the answers back to the task students have been working on so that the discussion remains meaningful. It is also helpful to pose more tasks using a similar format so that the students can apply what they have learned to new situations.

AN EXAMPLE OF A 'GOOD' QUESTION

Now let us have a look at how these steps would apply to the following 'good' question.

Two-fifths ($\frac{2}{5}$) of the students in a school borrow books from the library each day. How many students might there be in the school and how many of them borrow books each day?

STEP 1: POSE THE 'GOOD' QUESTION

Have the question written on the board and as you ask the question refer to the words on the board. Ask some students to read the question out loud and ask others to tell you what it means in their own words. Let students ask you any questions they may have. Explain the task to them if necessary but do not give any directions or suggestions on how to do the task. This is for the students to work out for themselves.

STEP 2: STUDENTS WORK ON THE 'GOOD' QUESTION

Organise the students to work in groups. Once they start working check that they are able to continue without teacher assistance. If necessary stop them and have a whole class discussion to overcome any general concerns. If lots of groups are finding it difficult, you could make the question easier by changing the fraction to a unit fraction such as $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{5}$, or suggest that the groups use counters to represent the school students. If only one or two groups are finding it difficult let them start on an easier related fraction such as $\frac{1}{5}$, and when they understand this extend it to $\frac{2}{5}$.

Monitor the progress of the groups. If a group stops after giving one response, ask them to look for other possible answers. If they have found a few answers you could ask them to think of a way to describe all their answers. For example, they could look for a pattern or a rule. You could also give a related task to extend them such as *Find the pattern if $\frac{3}{5}$ of the students borrow books each day.*

When all groups have at least one response to the question give them a five-minute warning and after this time stop all students. Do not be concerned that groups are at different stages.

STEP 3: WHOLE CLASS REVIEW

Ask the groups in turn to present their responses to the class. Some groups may want to use the counters to show their responses. Remember that students can respond at a variety of levels. For example, some possible responses are:

- It could be anything.
- 100 students, 40 of whom borrow books each day.
- The number of students in the school is a multiple of 5, such as 5, 10, 15, 20 and so on, and the number borrowing books would then be 2, 4, 6, 8 and so on.

These three responses differ not only in the level of mathematical understanding but also in the quality of thinking that is demonstrated by the answers. Try to take a positive approach to each group's response. For example, if the first response is given you could agree with the group and then ask them if they can give a specific answer. The group who gave the third response could be asked to demonstrate it using counters if they have not already done so.

STEP 4: TEACHER SUMMARY

The main points from the activity are the pattern that emerges (2:5, 4:10, 6:15, etc.), and the use of fractions as operators (e.g. $\frac{2}{5}$ of 10). Even if these points have been discussed it is important to go over them again. It would also be helpful to ask students to suggest how they would calculate $\frac{2}{5}$ of certain amounts and let them demonstrate using materials. You could also look at what happens to the answer when the amount is not a multiple of 5. As you are summarising do not lose sight of the original question. Refer to it when necessary to make a point.

A similar task that you could pose is *In a survey I found that $\frac{3}{4}$ of the people liked cooking. How many people did I ask, and how many liked cooking?*

Thus we can see that using 'good' questions in your classroom requires a different lesson format from a lesson in which the teacher demonstrates a technique or skill and follows up with student practice. It places different demands on a teacher too. As well as being receptive to all students' responses, the teacher must acknowledge the validity of the various responses while making clear any limitations, drawing out contradictions or misconceptions, and building class discussion from partial answers. We have seen how 'good' questions provide the environment for better learning; it is up to the teacher to ensure that the opportunities for learning become realities.

PART

TWO

USING ‘GOOD’ QUESTIONS AS PART OF RICHER ASSESSMENT

Earlier in this book we discussed how the manner in which students respond to 'good' questions can show the teacher if they understand the concept. Whereas students can often obtain correct answers to conventional tasks by merely remembering a rule or a procedure, their ability to respond to a 'good' question usually requires an understanding of the underlying principle or concept. Because such tasks have the potential to reveal what the students know rather than do not know, they provide rich information for the teacher and provide opportunities to extend student work.

By using 'good' questions as part of a classroom mathematics program teachers are thus able to collect richer assessment data about each student which they can then use to improve learning.

When collecting assessment data from students' responses to 'good' questions there are a couple of approaches that have proven to be both useful and effective. One of these approaches focuses on students' understanding of the mathematical content of a 'good' question and the other focuses on the behaviours displayed by students as they work at a 'good' question. Both approaches are explained in more detail below.

1 UNDERSTANDING MATHEMATICAL CONTENT

As it does not make sense to score students' responses to 'good' questions in terms of correct and incorrect answers, an alternative is to use a holistic method. This might involve sorting students' responses into various categories such as: 'no understanding evident'; 'made an attempt but shows limited understanding'; 'acceptable but incomplete'; 'demonstrates clear understanding'; and 'goes beyond expectations'.

In those cases where students have been required to give a written response to a 'good' question it is easy for teachers to collect the responses and sort them into these or similar categories. However, in situations where students are not required or are not able to give a written response, such as when concrete materials are being used, then teachers need another means of sorting their responses into categories. This can be achieved if teachers have a list of students' names and space to make comments next to each name. As teachers observe students at work they can note demonstrated understandings as they occur.

To give you a better idea of how the above method can be put into practice the following example shows how a year 1 teacher sorted some students' responses to this open-ended task:

Two clowns were each holding a bunch of balloons. One clown had 3 more balloons than the other clown. Draw some pictures to show how many balloons each clown might have been holding.

'no understanding evident': Included in this category was a picture drawn by one student of two clowns each holding three balloons. Also included was a picture of two clowns, one holding two balloons and the other holding one balloon.

'made an attempt but shows limited understanding': Included in this category was a picture of two clowns, one with 3 balloons and one with no balloons. When the student who drew the picture was asked to draw another possibility they insisted that there was no other possible answer.

'acceptable but incomplete': Included in this category was the work of a student who drew three pictures, each of two clowns, in which one clown was holding 3 more balloons than the other clown. However, the student showed no evidence of systematic working out. The pictures did not follow on from each other as regards the number of balloons. When discussing the pictures with the teacher the student did not demonstrate that they knew the possibilities could be worked out systematically.

'demonstrates clear understanding': Included in this category was a page of pictures of two clowns showing evidence of systematic working out. The first picture showed 3 balloons and no balloons, the second showed 4 balloons and 1 balloon … the last showed 10 balloons and 7 balloons.

'goes beyond expectations': Included in this category was the work of one student whose pictures showed a systematic approach to the range of possibilities. The first picture showed two clowns, one holding 3 balloons and the other none. The second picture was of two clowns, one holding 4 balloons and the other 1. The third picture was of two clowns, one holding 5 balloons and the other holding 2 balloons. The student also stated that it did not matter how many balloons the first clown had as long as the second clown always had 3 more than the first clown.

Collecting assessment data of this type assists teachers to find out what students know, what misconceptions (if any) they may have and it also allows teachers to make specific plans for future sessions. These descriptions of the categories are useful for communicating with the students and recording the assessment for later use.

2 DISPLAYED BEHAVIOURS

When students are working at a 'good' question it is an ideal time not only to assess their mathematical understanding but also to assess how they are working.

The following checklist items can be used as a guide to the types of behaviours we would expect students to demonstrate as they work at open-ended questions. Not all of these behaviours will be evident in every student. Nor will each student display all these behaviours at one time.

There are various ways these checklist items can be used. One way is to have a separate sheet for each student with the behaviours listed down one side or across the top and spaces to tick or date when a behaviour is observed. Another way is to have a class list with the behaviours listed across the top. In this case you could place a mark (/) next to a student's name when the behaviour is first observed and cross that mark (X) when the behaviour is consistently seen. Whatever way you choose it is a good idea to focus on one or two groups of students at any one time rather than the whole class.

- Works cooperatively
- Works independently
- Makes a plan
- Keeps trying
- When stuck tries something different
- Discusses work with others in group
- Uses materials when useful
- Draws diagrams when necessary
- Concentrates on the task
- Asks questions
- Organises information systematically
- Explains or displays ideas clearly
- Looks for all possibilities
- Able to generalise
- Accepts assistance from others
- Is confident
- Uses a range of mathematical strategies

PART

THREE

'GOOD' QUESTIONS TO USE IN MATHS LESSONS

This section contains many 'good' questions for you to select from and use in your classroom.

Questions are presented for 16 mathematics topics in the areas of Number and Algebra, Measurement and Geometry, and Statistics and Probability. The questions for each topic are organised into three levels:

Junior — beginning, years 1 and 2
Middle — years 3 and 4
Upper — years 5 and 6.

For the topic of decimals there are only questions for the Middle and Upper years as students should only experience decimals in an informal way in the Junior years.

At the beginning of each level is a list of experiences that students should encounter for the particular topic. Not all students will be ready for these experiences at the same time. It is quite possible that some students in the Middle years might be working on some of the experiences listed for the Junior years, while other students in the Middle years are working on some of the experiences listed for the Upper years. They should not be treated as a progression of experiences but rather as a range of possible experiences.

Many of the questions in these levels can be adapted to meet the needs of the students in your classroom by making them easier or more difficult.

As you are reading through the 'good' questions that follow, you will find some instances where they have been written as tasks rather than questions. This has been done where we felt they were better written as tasks. Use them in exactly the same way as the questions.

Below each question there are Teacher Notes. Sometimes these are to make you aware of some important teaching points for the particular question. They may also help you ascertain if students have understood the concept being presented. At other times they will be useful in helping you assess where students are at so you can plan to overcome any difficulties. It is a good idea to make notes as you observe students working so that you can use them in future planning.

A list of materials that you might need is provided at the beginning of each level. You will not need all of these materials unless you complete every question listed for the topic at that level. Check that you have suitable materials before you present a question to your class. It is important that students have a variety of concrete materials to select from when they are working on mathematical tasks.

NUMBER AND ALGEBRA

The six topics included in this strand are:

1 Money and Financial Mathematics
2 Fractions
3 Decimals
4 Place Value
5 Counting, Pattern and Order
6 Operations

There are links in these number and algebra topics with the other areas of the mathematics curriculum and with each other. It is neither possible nor useful to try to treat them separately. The questions in each topic do, however, have their main teaching point within that topic.

While answering these questions, students will develop a feeling for the way numbers work. They will develop a strong sense not only for whole numbers but also for where fractions and decimals fit into the number system. They will understand the importance of estimation and mental calculation skills and use calculators to enable them to understand key ideas without having to do complicated calculations before they are ready to do so.

Do not forget to adapt questions where necessary by making numbers or amounts smaller or larger.

MONEY AND FINANCIAL MATHEMATICS (JUNIOR)

EXPERIENCES AT THIS LEVEL WILL HELP STUDENTS TO:

- recognise different coins and notes
- describe, sort and classify coins and notes
- exchange money for goods in play situations and give appropriate change
- order money amounts
- use coins to represent written money amounts and use numbers to record the value of a group of coins
- use estimation and a calculator for money calculations.

MATERIALS NEEDED:

- play money, coins and notes
- ‘goods’ marked with varying prices (ensure that there are combinations of items that add to $1)

‘GOOD’ QUESTIONS AND TEACHER NOTES

1 **How many different ways can you make 20c?**

2 **In my pocket I have 75c. What coins might I have?**
Q1 & Q2 Students should realise that there are many different ways to make a money amount. See if they use multiples of one coin, e.g. 4 × 5c, as well as different coins, e.g. 10c + 5c + 5c. Notice if they develop a systematic way to find coin combinations, e.g. for 75c they might start with 15 × 5c, then 7 × 10c + 5c, then 3 × 20c + 10c + 5c, etc.
Are students confident when counting in 5s, 10s, 20s?

3 **Choose an amount of money, for example, 75c, $1.20 … Ask students to choose an item from some items on display that they can afford to buy with the amount of money. Ask them to choose an item that they cannot afford to buy with the amount of money.**
You will need a variety of items each with a price tag showing.

4 **I bought something at a supermarket and got 5c change. How much did it cost and how much money did I give to pay for it?**
Students should develop a system. Some may do it randomly to begin with and then realise the pattern, e.g.:
- costs 5c and gives 10c
- costs 15c and gives 20c
- costs 95c and gives $1
- costs $1.95 and gives $2.

Can students see the folly of giving 15c for an item costing 10c to receive 5c change?

5 **I spent exactly $1 at our class shop. What might I have bought?**
Check how easily students can add amounts to $1. They should be able to mentally estimate by rounding before writing down amounts.
Note if they calculate multiples of 5, 10 or 20 to make $1, e.g. do they know five items at 10c each is 50c or do they add each one separately?

6 **I am a coin with an animal on me that does not swim. What might I be?**
The main focus here is to look more closely at the attributes of coins.

7 **I have two coins in one hand and one in the other hand. Each hand is of equal value. What could the coins be?**
Note if students develop a system when recording. How easily do they calculate amounts?

8 **The answer to a calculation is 75c. What might the question be? Refer to this list to help you.**

CANTEEN PRICE LIST

Vegemite sandwich	$2.10	Salad sandwich	$2.60
Ham & salad roll	$2.90	Bag of chips	$1.25
Fruit salad	$2.15	Piece of fruit	50c
Cookie	25c		

Can students write more than one question? Do they limit their questions to addition or do they make others?

9 I had one of each of the coins in our currency on my table. I sorted them into two groups. What might the groups have been?
It is interesting to note what categories students use. Ask them to tell you their categories; don't assume you know their reasoning.

10 If I have three Australian coins in my pocket, how much money might I have?
Ask students to be systematic, and to find out how many different amounts they might have.

11 The price tag on a toy car is $2.75. What coins would I use to pay for this?
Note if students develop a system when recording. How easily do they calculate amounts?

12 I have exactly $100 in notes in my pocket. What notes might I have?
Check how easily students can count in 5s, 10s, 20s, 50s.
Are they aware of available notes? Are they systematic in recording?

13 Someone was asked to remember the cost of five items. They knew the most expensive was $2 and the least expensive was 50c. What might the other three be?
The focus here is on ordering of money amounts. Note if the students record different amounts correctly.

MONEY AND FINANCIAL MATHEMATICS (MIDDLE)

EXPERIENCES AT THIS LEVEL WILL HELP STUDENTS TO:

- round to the nearest dollar to estimate or check total cost
- record money amounts
- tender appropriate amounts when the exact amount is not available
- order money amounts
- use an appropriate method (mental, written, calculator) to solve problems involving money and processes.

MATERIALS NEEDED:

- play money, notes and coins
- supermarket catalogues
- calculators.

'GOOD' QUESTIONS AND TEACHER NOTES

1 I bought an item at a shop and got 35c change. What did I buy and how much did it cost?
Students need to see the folly of including such things as buying an item costing 5c and giving 40c to get 35c change.
Note if students look for a pattern when recording answers.

2 I gave change of $1 using 20c, 10c and 5c coins. What might the change have looked like?
Note if students record systematically and accurately.
How easily do they deal with addition of 20, 10 and 5 and multiplication of each of these to make them up to $1?

3 Show some different ways to give change from $2 for an item costing $1.35.
Allow students to use play coins. Do they systematically record the different ways?

4 How could I spend exactly $20 at the supermarket? (Use a catalogue and a calculator to help.)
Check if students use estimation skills to help them. They might round off some amounts to assist their estimation. Note how they use the calculator.

5 In my pocket I have $36. What notes and coins might I have?
This allows you to see how familiar students are with the various notes and coins and if they use a system when recording.

6 If I have three Australian notes in my wallet, how much money might I have?
Ask students to be systematic, and to find out how many different amounts they might have.

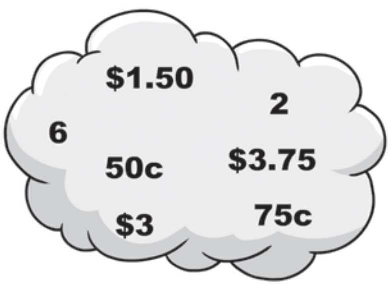
7 I paid $60 for six tickets to a school concert. How many adults and children are there and how much is each ticket?
Are the answers realistic? Can students multiply amounts, e.g. $4 \times \$10$ or $4 \times \$5$? Note how they do this, i.e. mentally, with a calculator, etc.

8 I looked in the paper and saw five cars. The most expensive was $12 150, the cheapest was $9700, and the one in the middle was $10 000. What prices might the two missing cars be?
The focus here is on ordering of money amounts. How easily can students work with numbers above and below 10 000?

9 When I was in a gift shop I saw that a plate cost about $22 and a bowl about $15. What might have been the price tag on the plate and the bowl?
This question focuses on rounding off. Are students aware that they can round up and down?

10 Prices in supermarkets are to the exact cent even though we do not have 1c and 2c coins. I bought two items and was asked to pay $4.65. What might have been the price of each item?
See if students realise that the two amounts do not have to total $4.65 exactly, i.e. one could be $1.22 and the other $3.41, which totals $4.63 and rounded off makes $4.65.

11 A number sentence uses three of the amounts or numbers in this cloud. What might the number sentence be?
The main focus here is to see if students use a variety of processes, e.g. $6 \times 50c = \$3$, $\$1.50 \div 2 = 75c$, $\$3.75 - 75c = \$3$.



12 My friends and I shared an amount of money equally among us. We each got $1.20. How much money was there and how many friends might I have?
It is interesting to see how students do this — mentally, by writing down or with coins. When they check their answer do they include themselves or only the friends?

13 I bought something and paid for it with three coins. What might it have been and how much did it cost?
Look for a range of responses that are realistic.

14 I went to get $100 out of the bank. What are the different ways I can ask for this amount of notes?
How easily can students multiply and divide by 2, 5, 10 and 20?

MONEY AND FINANCIAL MATHEMATICS (UPPER)

EXPERIENCES AT THIS LEVEL WILL HELP STUDENTS TO:

- use mental calculation and estimation
- use +, −, × and ÷ for written computation of money
- select an appropriate operation to solve problems involving money.

MATERIALS NEEDED:

- play money, notes and coins
- new and used car section of a newspaper (or similar)
- current exchange rates
- calculators

'GOOD' QUESTIONS AND TEACHER NOTES

1 Scientific calculators cost $40 and basic calculators cost $12. How much might it cost for a class set of some basic and some scientific calculators?
Note how students decide how many of each calculator to purchase. Do they record their answers systematically? Do they choose appropriate operations to work out the price? How easily do they handle these operations?

2 I have $18 000 and want to buy two cars. What could I buy?
Note if students can justify their answers and if they can provide a range of answers.

3 If one of the notes currently in use was to be changed to a coin, which one would you choose? Why?
Students should be able to justify their choice in a reasonable manner. You could extend this by looking at notes and coins in use in other countries.

4 **You are spending five nights away. You have won $1000 for accommodation. Where could you stay?**

Top class hotel	$600 per night
4 star hotel	$450 " "
3 star hotel	$200 " "
2 star hotel	$120 " "
Backpackers	$50 " "

Note what methods students use to work this out, i.e. do they readily multiply amounts when needed or do they always add amounts? They can stay at different places.

5 **Imagine that you are in some other country. A hamburger costs 10 units of that country's money. What country might you be in?**
Students can use the web or newspaper to find out information about exchange rates.

6 **Design a rounding policy for a supermarket.**
The way students approach this will tell you if they understand rounding. It is interesting to note whose side they are on — owner or customer?

7 **I have more than $10 in coins, but I cannot change a $10 note into coins exactly. How much money might I have?**
It is a good idea to let students use play coins. This is a hard question. You could hint that when we used 1c and 2c coins, if you had 3 × 2c coins you could not change 5c.